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NUMERICAL SOLUTION OF AN ILL-POSED PROBLEM ARISING IN WIND TUNN--ETC(U)
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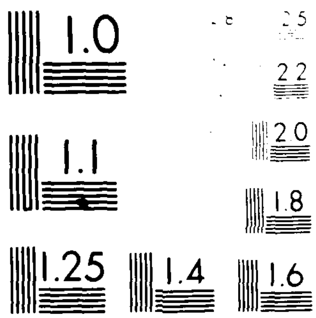
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NSWC TR 82-32

NUMERICAL SOLUTION OF AN ILL-POSED PROBLEM ARISING IN WIND TUNNEL HEAT TRANSFER DATA REDUCTION

BY JOHN B. BELL, ANDREW B. WARDLAW

RESEARCH AND TECHNOLOGY DEPARTMENT

4 DECEMBER 1981

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report documents a computer program which solves an improperly posed heat conduction problem arising in the measurement of heat transfer data in wind tunnel experiments. The mathematical model which is treated is one dimensional and assumes constant material properties. The computational approach is based on an adaptation of a method used by Alifanov to treat a similar problem. The Green's function for a well posed heat conduction problem is used to reduce the problem to a Volterra integral equation of the first kind. Tikhonov

regularization techniques are then applied to compute stable approximate solutions. Numerical results for several cases typical of a class of wind tunnel experiments are presented.

FOREWORD

This report documents a computer program which solves an improperly posed heat conduction problem arising in the measurement of heat transfer data in wind tunnel experiments. The mathematical model which is treated is one dimensional and assumes constant material properties. The computational approach is based on an adaptation of a method used by Alifanov to treat a similar problem. The Green's function for a well posed heat conduction problem is used to reduce the problem to a Volterra integral equation of the first kind. Tikhonov regularization techniques are then applied to compute stable approximate solutions. Numerical results for several cases typical of a class of wind tunnel experiments are presented.

Ira M Blatstein

IRA M. BLATSTEIN
By direction

CONTENTS

ILLUSTRATIONS

1. INTRODUCTION

One of the aerodynamic properties to be measured during a wind tunnel experiment is the heat transfer rate to the surface of a thin walled wind tunnel model. Traditionally this is done by assuming a functional form for the temperature profiles and using measured data to fit parameters. Such approximations introduce significant error in short duration wind tunnel tests which feature rapid model pitching. Unfortunately, improving upon this simple approximation is not straightforward because the mathematical problem describing the relationship between the measured data and the desired information is improperly posed and is consequently unstable with respect to small measurement errors.

Various treatments of ill-posed heat conduction problems have appeared in the literature. An excellent survey of engineering methods can be found in Beck.[1] Somewhat more mathematical approaches can be found, for example, in Cannon and Douglas[2] and in Ginsberg.[3]

The approach used in this report is a modification of a method used by Alifanov[4] to treat a similar problem. Essentially, the method recasts the problem as an integral equation of the first kind which is ill-posed. A Tikhonov regularization procedure[5] is then used to compute stable approximate solutions to the integral equation.

In the next section, the mathematical model is presented and the integral form of the solution is derived from the Green's function for the heat equation. The application of the Tikhonov regularization procedure and the subsequent discretizations are discussed in Section 3. The last section presents some numerical results characteristic of wind tunnel experiments. A listing of a program implementing the method is given in Appendix A.

2. MATHEMATICAL FORMULATION

To determine heat transfer rates on a body in a wind tunnel experiment it is necessary to measure heat flux on the surface of the model. However, it is often inconvenient or excessively costly to place thermocouples directly on the surface of the model which do not effect flow characteristics. Instead it is common practice to place thermocouples on the inside surfaces of thin walled models which results in a direct measurements of the inner surface temperature. If we assume constant material properties and that heat conduction along the body surface can be ignored, the mathematical problem of determining heat transfer rates on the front surface reduces to determining

$$\kappa U_{x'} (\ell, t')$$

given that the temperature U (x', t') satisfies

$$\rho c\, U_{t'} = \kappa U_{x'x'} \qquad 0 < t' \leq T, \quad 0 < x' < \ell$$

$$U(0,t') = \hat{f}(t') \qquad 0 < t' \leq T$$

$$U(x',0) = 0 \qquad 0 < x' < \ell$$

$$\kappa U_{x'}(0,t) = 0 \qquad 0 < t' \leq T$$

Here t' denotes time and x' corresponds to the distance from the backface with $x' = 0, \ell$ representing the inner and outer surfaces respectively. Also, ρc is the volumetric heat capacity and κ is the thermal diffusivity. The function $\hat{f}(t')$ respresents the measured temperature data.

The two additional boundary conditions $U(x',0) = 0$, $\kappa U_{x'}(0,t) = 0$ are reasonable approximations of typical wind tunnel conditions. The first assumes that the initial temperature is constant (normalized to zero) while the second presumes an adiabatic inner surface. This second assumption is reasonable because of the extremely low pressure inside the model during a test and the short duration

of the tests under consideration. Note, however, that the more general case where $u(x,0)$ and $\kappa\, U_x(0,t)$ are known functions can be reduced to the above form by solved a well posed auxiliary heat conduction problem.

The nondimensional form of the problem obtained by setting $t = t'/T$, $\alpha = \rho c_p/\kappa$ and $x = x'/\ell$ is to find

$$\alpha\, U_x(1,t) \qquad\qquad 0 < t \leq 1$$

given that U satisfies

$$\begin{aligned} U_t &= \alpha\, U_{xx} && 0 < t \leq 1, \quad 0 < x < 1 \\ U(x,0) &= 0 && 0 < x < 1 \\ U(0,t) &= \hat{f}(t) && 0 < t \leq 1 \\ U_x(0,t) &= 0 && 0 < t \leq 1 \end{aligned} \tag{1}$$

In applying the Tikhonov regularization techniques to the problem (1) it is helpful to recast the problem as an integral equation. We suppose that we are given $\alpha\, U_x(1,t) = g(t)$ and consider the problem

$$\begin{aligned} U_t &= \alpha\, U_{xx} \\ U(x,0) &= 0 \\ U(0,t) &= 0 \\ \alpha\, U_x(1,t) &= g(t) \end{aligned} \tag{1'}$$

The solution to this problem can be written (cf. Ref. 6) in the form

$$U(x,t) = \int_0^t \Theta(x,t-\tau)\, g(\tau)\, d\tau$$

where

$$\Theta(x,t) = \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{\pi\alpha t}} \exp\left[\frac{-(x+2n)^2}{4\alpha t}\right]$$

Using this integral representation the problem (1) reduces to finding g(t) such that

$$f(t) = \int_0^t \Theta(0,t-\tau)\ g(\tau)d\tau \equiv \ Kg \qquad (2)$$

Unfortunately the integral equation (2) for g; reflecting the ill-posed nature of the original problem, is of first kind; consequently, small perturbations in $\hat{f}$ cause large changes in the corresponding g(t). In fact, since $\hat{f}$ contains measurement errors, it will not be smooth and, in general, there will be no $g \varepsilon L^2$ (square integrable) with satisfies the operator equation

$$Kg = \hat{f}.$$

Nevertheless, special techniques can be used to compute stable approximate solutions to be integral equation.

3. REGULARIZATION AND NUMERICAL APPROXIMATION

In the previous section we used an integral representation of the solution of (1) to reduce the problem to a first kind integral equation of the form (2). We also noted that measurement error leads to possible nonexistence and instability. To overcome the nonexistence problem we note that for the correct g there is an f for which

$$Kg = f.$$

We now assume that we have a bound ε on the error introduced by measurement of the form

$$\int_0^1 (f(t) - \hat{f}(t))^2 \, dt \leq \varepsilon^2 \tag{3}$$

Based on the consideration that the true temperature profile at $x = 0$ is near $\hat{f}$ we seek a smooth g ($g \in H^1$, i.e. g and g' are square integrable) which minimizes

$$||Kg - \hat{f}||_{L^2} \tag{4}$$

However, the least squares form (4) still suffers instability properties. To stabilize the least squares problem (4) we use regularization as described in Tikhonov and Arsenin.[5] The idea behind regularization is to add a penalty to the least squares problem based on a norm of g. In this case we use H^1 - regularization of the form

$$\min \; ||Kg - \hat{f}||^2_{L^2} + \beta \; ||g||^2_{H^1} \tag{5}$$

where β is known as the regularization parameter. According to theory developed in [5], if $\beta = \beta(\varepsilon)$ is choosen so that the minimizer g_β of (5) satisfies

$$C_2 \; \varepsilon \leq ||Kg_\beta - \hat{f}||_{L^2} \leq C_1 \; \varepsilon \tag{6}$$

for two constants C_1 and C_2 then

$$g_\beta \to \alpha \; U_x(1,t) \text{ as } \varepsilon \to 0.$$

Thus, the computation of stable approximate solutions to the integral equation (2) reduces to the minimization of (5), suitably adjusting β to satisfy (6).

To discretize the minimization (5) we will restrict our consideration to a

finite dimensional set of functions S. We will then define a method of associating a function in S with Kg for $g \in S$. Since $\hat{f}$ is only given by discrete measured values, the most natural choice for S is the piecewise linear functions on a given grid. More precisely, we subdivide the time interval [0,1] into N subintervals of length $\Delta t = 1/N$ and let S be the set of functions which are continuous and linear on each subinterval. (The program assumes that N corresponds to the measured data points for $\hat{f}$ and that $\hat{f}$ is the piecewise linear interpolant of these measured values).

The minimization (5) has now become finite dimensional; but, we have yet to specify how to associate a function in S to Kg. One possible approach to this problem would be to approximate the integral in (2) by a quadrature rule and use this to define nodal values for Kg. Unfortunately the expression for Θ as an infinite sum makes this a cumbersome task.

A better approach is to recall that Kg is u(0,t) for the solution to the heat conduction problem (1'). Using this property and the linearity of K allows one to use a numerical solution procedure for (1') to find approximate nodal values for Kg. More precisely we define $g_\delta \in S$ to be the piecewise linear function such that

$$g_\delta\,(j\Delta t) = \begin{cases} 1 & \text{if } j = 1 \\ 0 & \text{if } j \neq 1 \end{cases}$$

Introducing a spatial finite difference grid and using the Crank-Nicholson integration scheme (cf. Ref. 7) we can solve (1') numerically with $g = g_\delta$ to obtain a vector of values $\vec{f}_\delta$ corresponding to the values of u $(0,i\Delta t)$. The linear interpolant f_δ of the nodal values $\vec{f}_\delta$ then approximates Kg_δ. For most applications the error introduced by the numerical integration procedure is much smaller then ε and can be ignored.

We now denote the components of $\vec{f}_\delta$ by $f_{\delta i}$ and define the matrix

$$A = \begin{pmatrix} f_{\delta_1} & & & & \\ f_{\delta_2} & f_{\delta_1} & & & \\ f_{\delta_3} & f_{\delta_2} & \ddots & & \\ \vdots & \vdots & \ddots & \ddots & \\ f_{\delta_N} & f_{\delta_{N-1}} & \cdots & f_{\delta_2} & f_{\delta_1} \end{pmatrix}$$

Using the linearity of K, superposition and denoting by $\vec{g}$ the vector of nodal values of g, we see that $A\vec{g}$ defines a vector which approximates nodal values of Kg.

To complete the discretization we note that for $g \in S$ we can evaluate the H^1 - norm to obtain

$$||g||^2_{H_1} \equiv \int_0^1 |g(t)|^2 + |g'(t)|^2 \, dt$$

$$= \vec{g}^T B_o \vec{g} + \vec{g} B_1 \vec{g} \equiv \vec{g} B_g$$

where

$$B_o = \frac{h}{3} \begin{pmatrix} 2 & \frac{1}{2} & & & \\ \frac{1}{2} & 2 & \frac{1}{2} & & \\ & \frac{1}{2} & 2 & \frac{1}{2} & \\ & & \ddots & \ddots & \frac{1}{2} \\ & & & \frac{1}{2} & 2 \end{pmatrix} \qquad B_1 = \frac{1}{h} \begin{pmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ & \ddots & \ddots & \ddots \\ & -1 & 2 & -1 \\ & & -1 & 1 \end{pmatrix}$$

and $B = B_1 + B_2$.

The minimization problem (5) now becomes

$$\min (A\vec{g} - \vec{f})^T (A\vec{g} - \vec{f}) + \beta \vec{g}^T B \vec{g}. \tag{7}$$

(Note that $(A\vec{g} - \vec{f})^T (A\vec{g} - \vec{f})$ is not the exact integral of the associated piecewise linear function; but, is only the sum of the squares of the residuals. This reflects the type of estimate available for ε whereas a higher order integration would only integrate the noise more precisely. The minimizer of $\vec{g}_\beta$ of (7)

ıtisfies the Euler equations; i.e.,

$$(A^T A + \beta B) \vec{g}_\beta = A^T \hat{\vec{f}} \tag{8}$$

ınce $A^T A + \beta B$ is symmetric and positive definite, (8) is uniquely solvable and $_\beta$ can be found by Gaussian elimination (cf. Ref. 8).

The value of β remains to be specified. The theoretical basis for this election is the discrepancy principle [5] of setting β according to (6). ı practice, since the residual

$$r(\beta) = \left[(A\vec{g}_\beta - \hat{\vec{f}})^T (A\vec{g}_\beta - \hat{\vec{f}})\right]^{\frac{1}{2}}$$

s a montone (increasing) function of β, β can be set by trial and error. icking β so that $r(\beta)$ was very close to ε was found to yield satisfactory results. umerical experiments indicated that once an appropriate β was selected, it need ot be changed unless the character of the noise changes.

4. NUMERICAL RESULTS

To test the algorithm three cases typical of a class of wind tunnel experiments were treated. In each case the length of the test was assumed to be 1.25 seconds and the material was assumed to be .054 inch thick steel with thermal conductivity $k = 0.00289$ Btu/ft. sec. ^{o}F and volumetric heat capacity $\rho c = 58.98$ Btu/ft^3 ^{o}F. To obtain data, three heat flux distributions were specified and the heat conduction equations were used to compute numerically the temperatures on the back face ($x = 0$). It should be noted that the assumption of linearity, one dimensionality and insulated boundary at $x = 0$ were assumed in obtaining the data; consequently, these numerical experiments do not test the validity of these assumptions. The specified heat fluxes and associated temperature profiles at $x = 0$ are shown in Figures 1a,1b and 1c.

We now have three "true" backface temperature profiles for which we know the associated heat fluxes. The numerical test will now be to add noise to the backface temperature distributions and attempt to recover the associated heat fluxes. Two noise levels were used. They are shown in Figures 2a and 2b. The first is typical of wind tunnel thermocouple measurement noise and the second prescribes a noise level much higher than expected in practice.

For the low noise level case, it was found that for $\beta = 10^{-4}$ the computed residual rms noise level agreed well with the true rms noise level. The numerical results for the three cases are shown in Figures 3a, 3b and 3c. The true solution is represented by the solid line and the computed solutions is shown by the dotted line.

Note that in each case there is poor agreement between true and computed solutions near the end of the interval. The cause of this problem is that the heat flux near the final time has very little effect on temperatures at $x = 0$. For this reason the second term in (5) dominates the minimization. The dominant term than

becomes

$$\int_0^1 |g'(\tau)|^2 \, d\tau$$

so that $g'(t) \to 0$ as $t \to T$.

For the high noise case the appropriate value of the regularization parameter was found to be $\beta = .001$. In Figures 4a, 4b and 4c we show, for each case, the noisy input temperature profiles (dotted line) compared with the true profiles and, as before, the computed heat fluxes.

Although the method is relatively insensitive to the specific value of β, care should be taken to insure that it is of the right order of magnitude. To illustrate this, the first case for low noise was repeated with β much to large (1) and β much too small (10^{-8}). These results are shown in Figures 5a and 5b respectively.

5. CONCLUDING REMARKS

The method described in this report provides an effective tool for computing heat fluxes from thermocouple data in wind tunnel experiments. For three typical cases with characteristic experimental noise levels excellent agreement with the known solution was obtained. The method is not completely automated and requires user judgement in its application. The user must specify a β value based on the expected noise level and often a trial and error procedure is required to optimize results. Also the regularization technique leads to degradation of results as $t \rightarrow T$. This problem can be circumvented by recording data past the end of the run. Finally, in the straightforward extension to the nonlinear case and multidimensional problems the method becomes extremely cumbersome and costly. However, the general approach seems promising and further work may make it possible to circumvent the above difficulties.

a) Case A

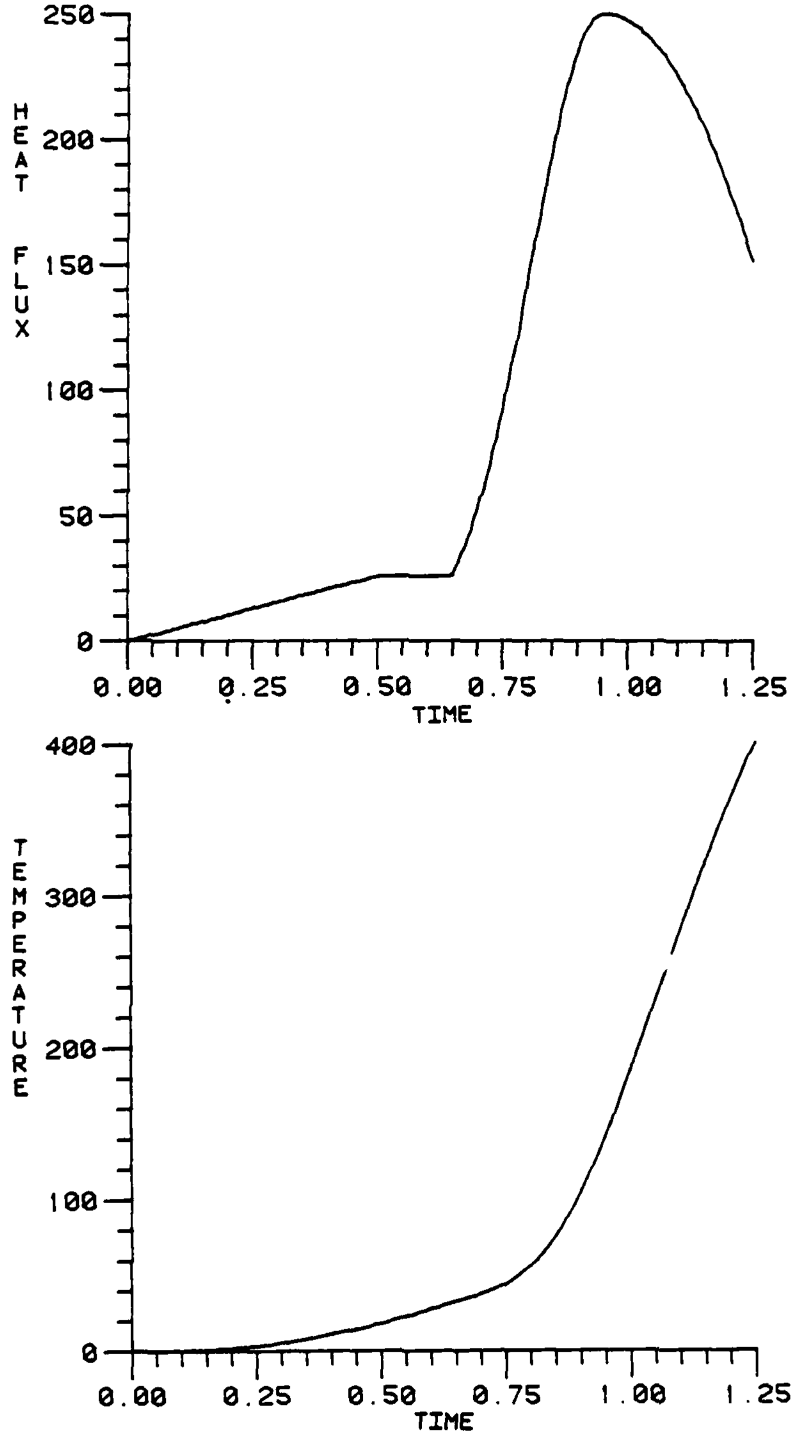

FIGURE 1 SPECIFIED HEAT FLUXES AND CORRESPONDING BACKFACE TEMPERATURE FOR THREE SAMPLE PROBLEMS ARE SHOWN IN PARTS A, B AND C

b) Case B

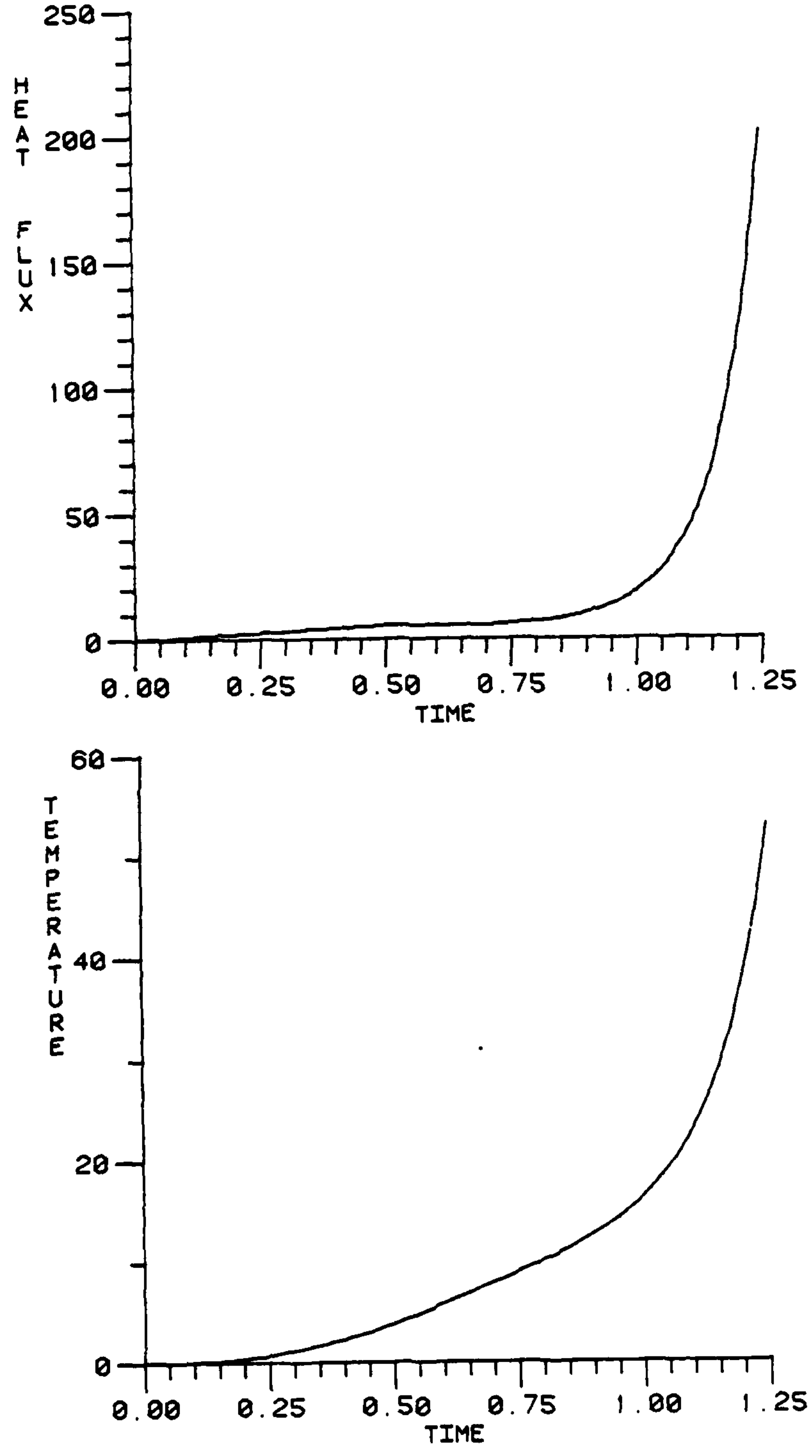

FIGURE 1 (CONTINUED)

c) Case C

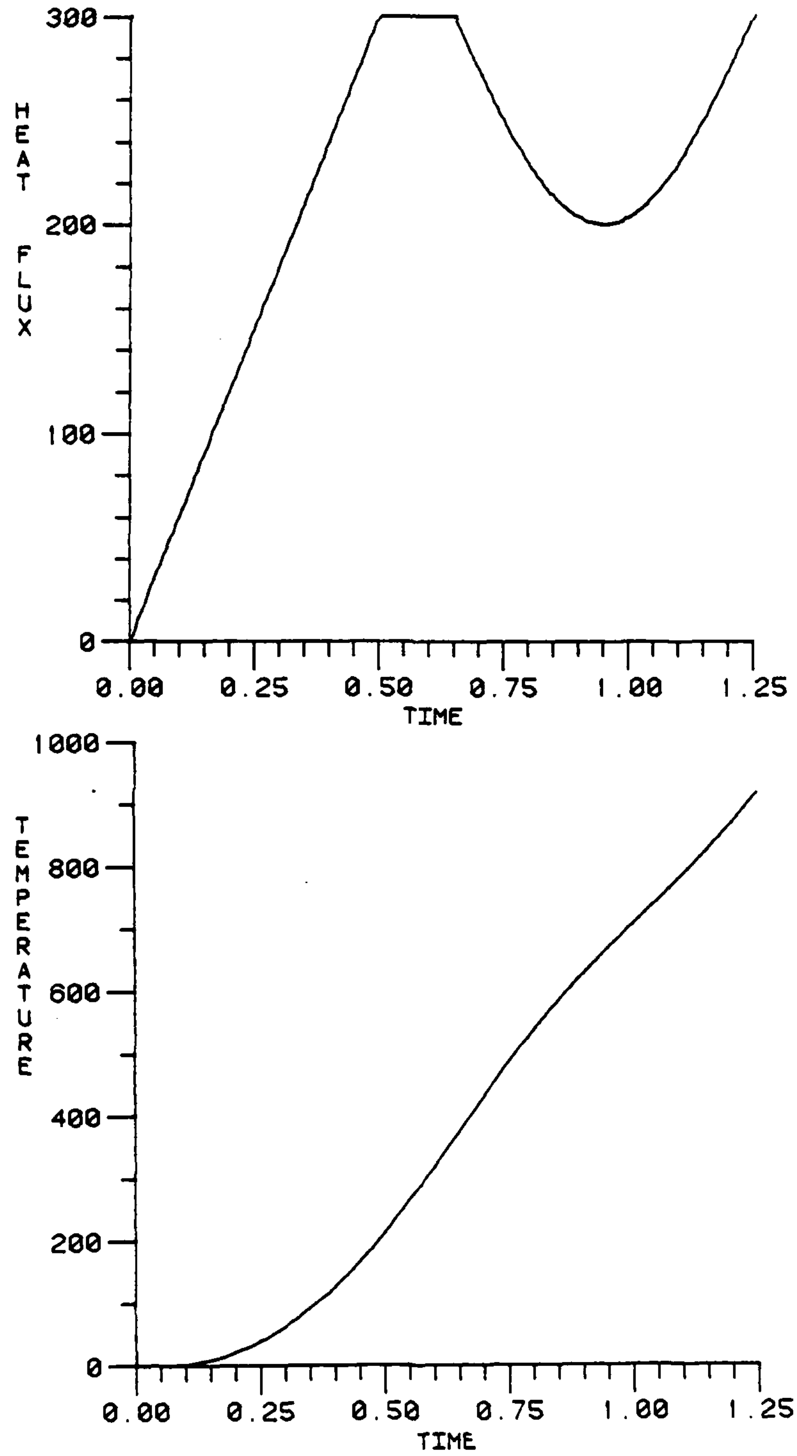

FIGURE 1 (CONTINUED)

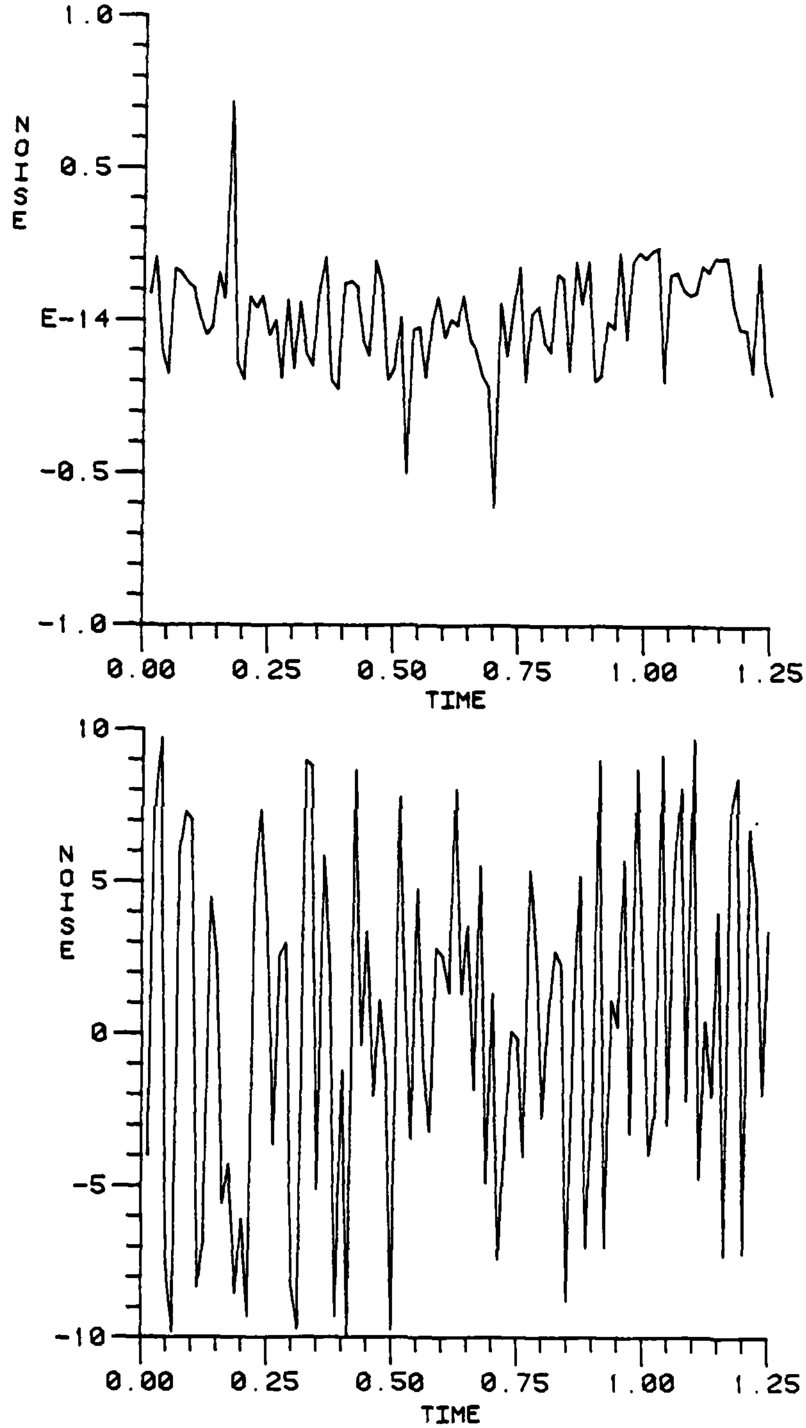

FIGURE 2 EXPECTED AND HIGHER THAN EXPECTED NOISE LEVELS

a) Case A

b) Case B

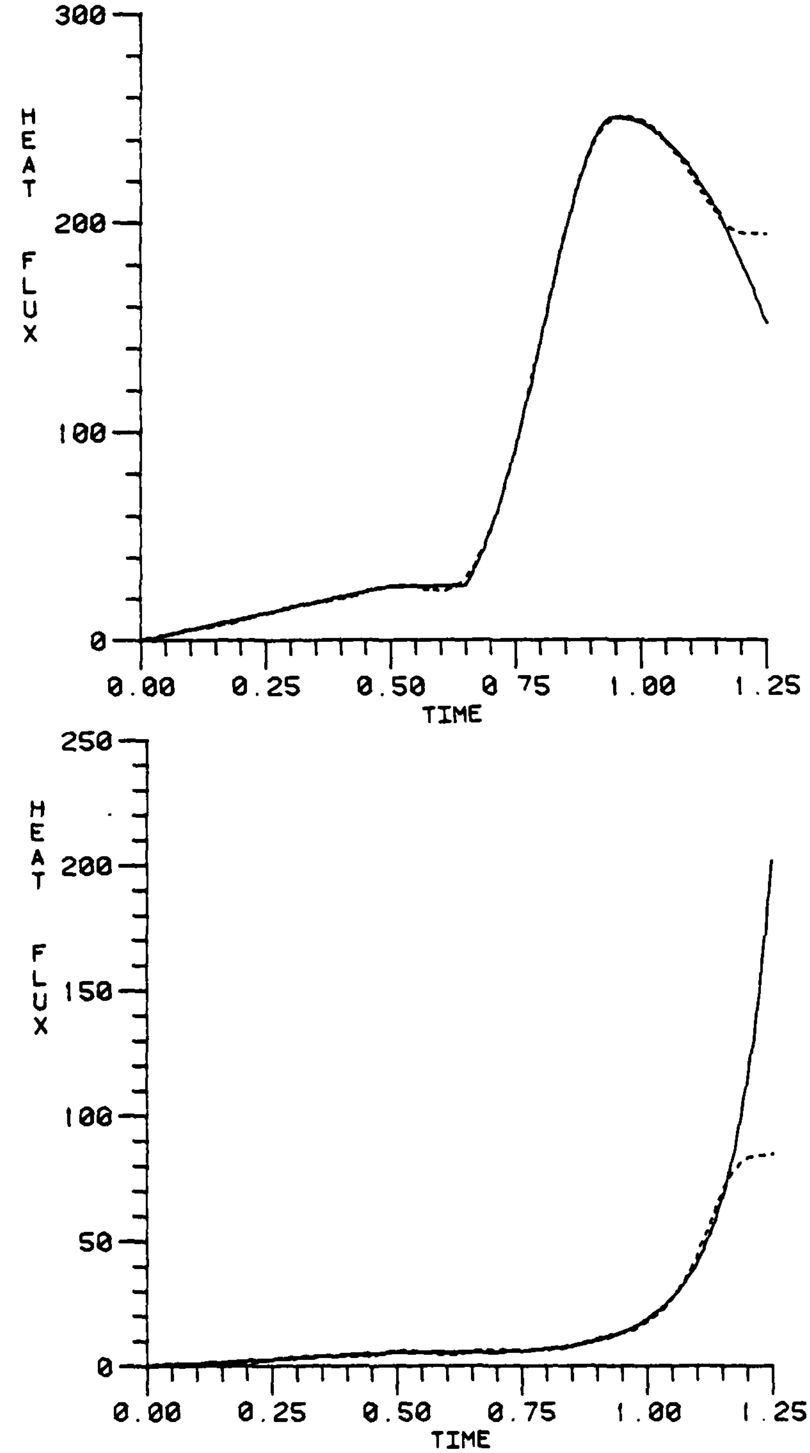

FIGURE 3 COMPARISON OF COMPUTED AND "TRUE" BACKFACE TEMPERATURES FOR CASES A, B AND C USING EXPECTED NOISE LEVELS

c) Case C

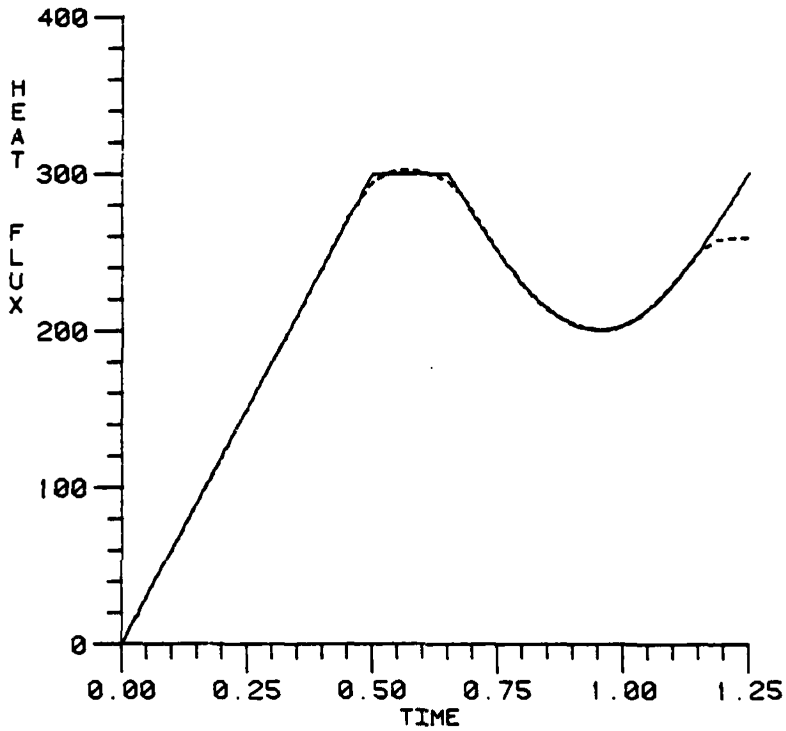

FIGURE 3 (CONTINUED)

a) Case A

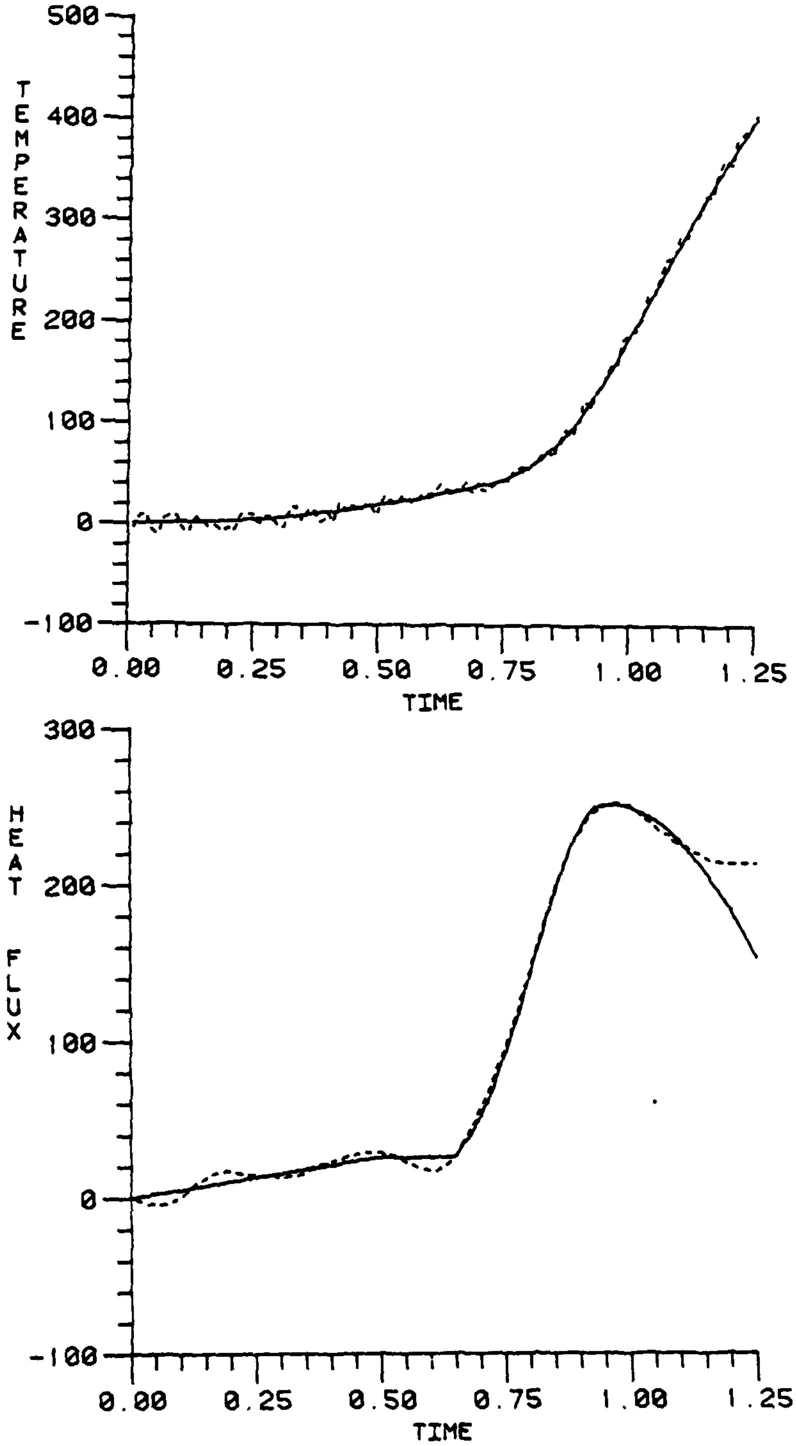

FIGURE 4 COMPARISON OF COMPUTED AND "TRUE" BACKFACE TEMPERATURES FOR CASES A, B AND C USING HIGHER THAN EXPECTED NOISE LEVELS

b) Case B

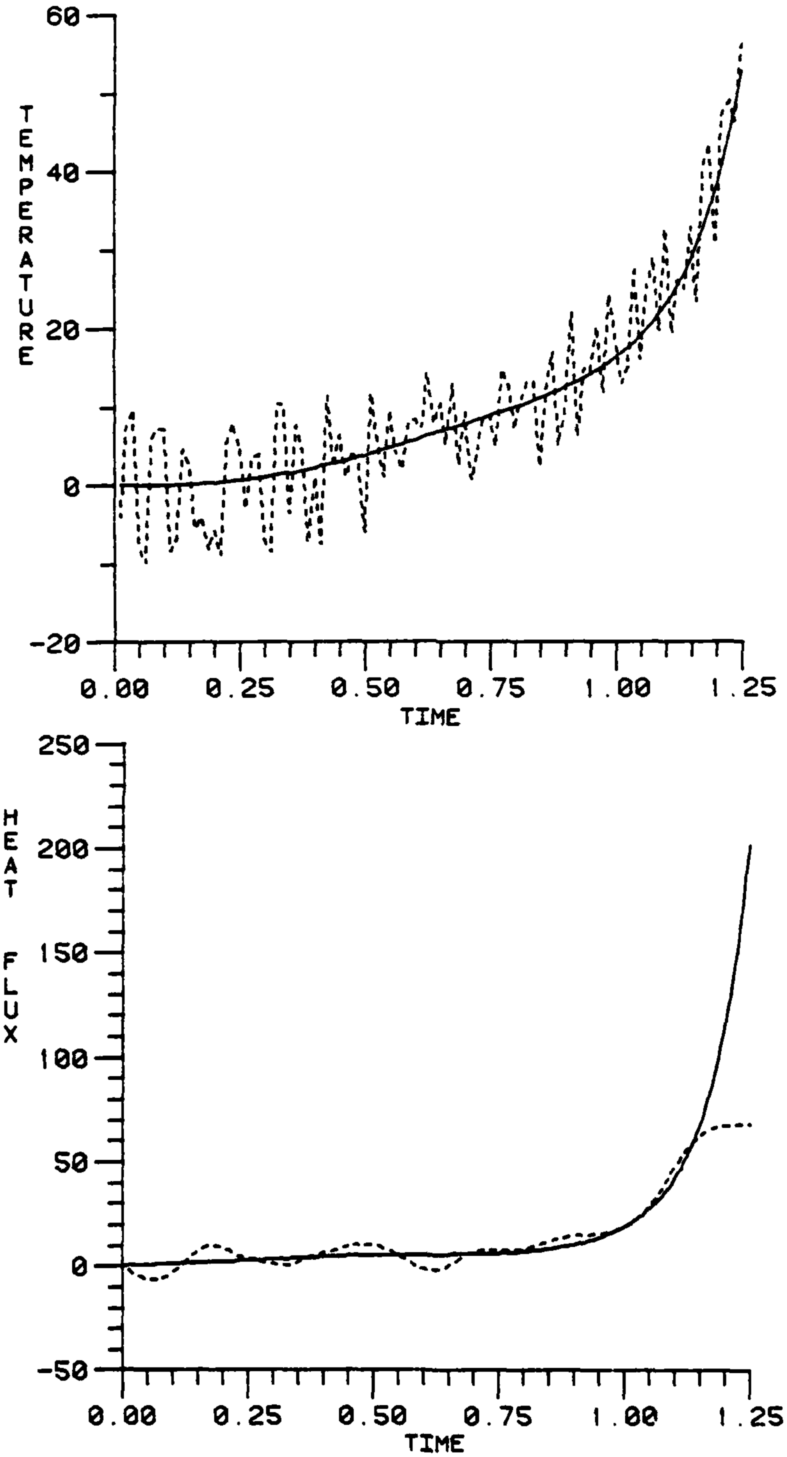

FIGURE 4 (CONTINUED)

c) Case C

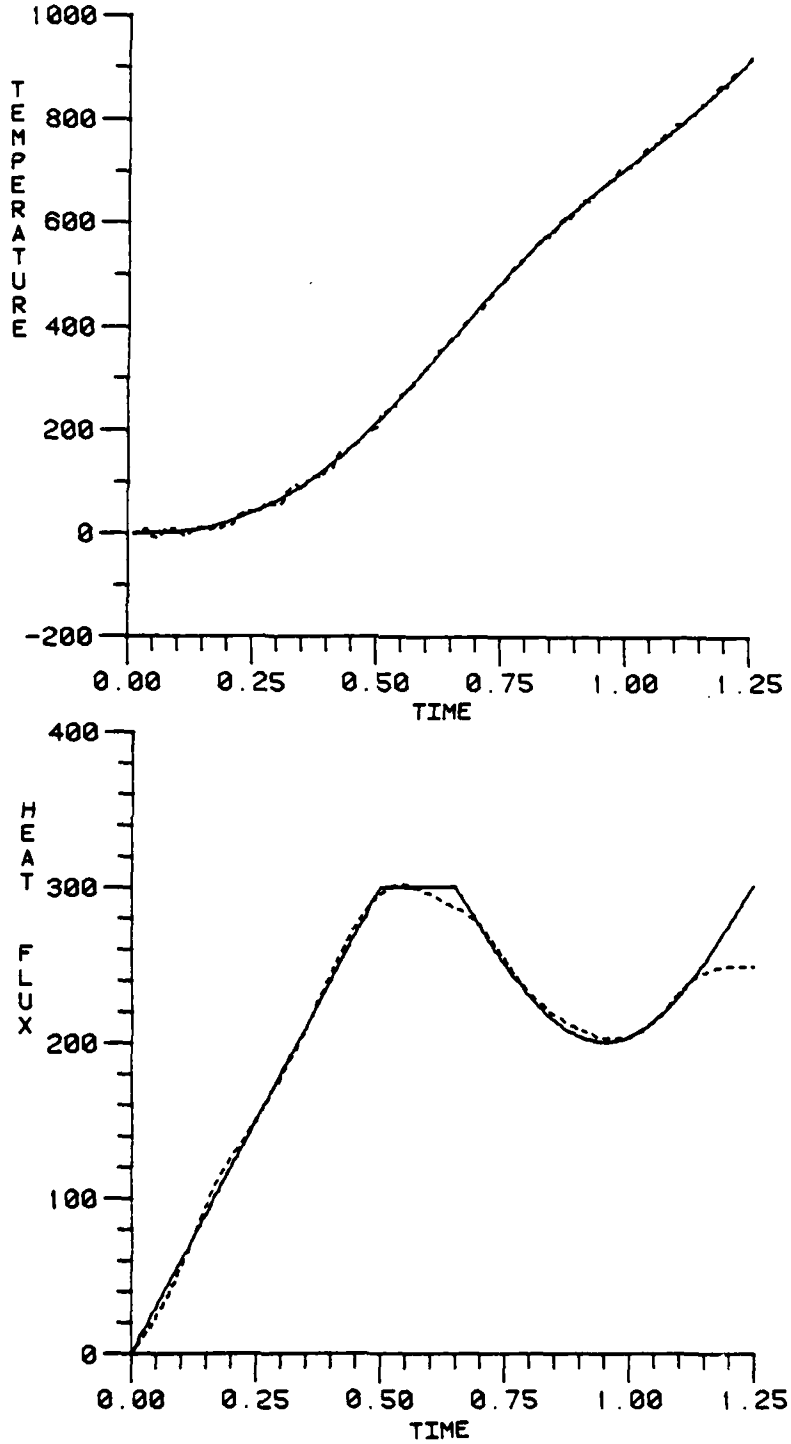

FIGURE 4 (CONTINUED)

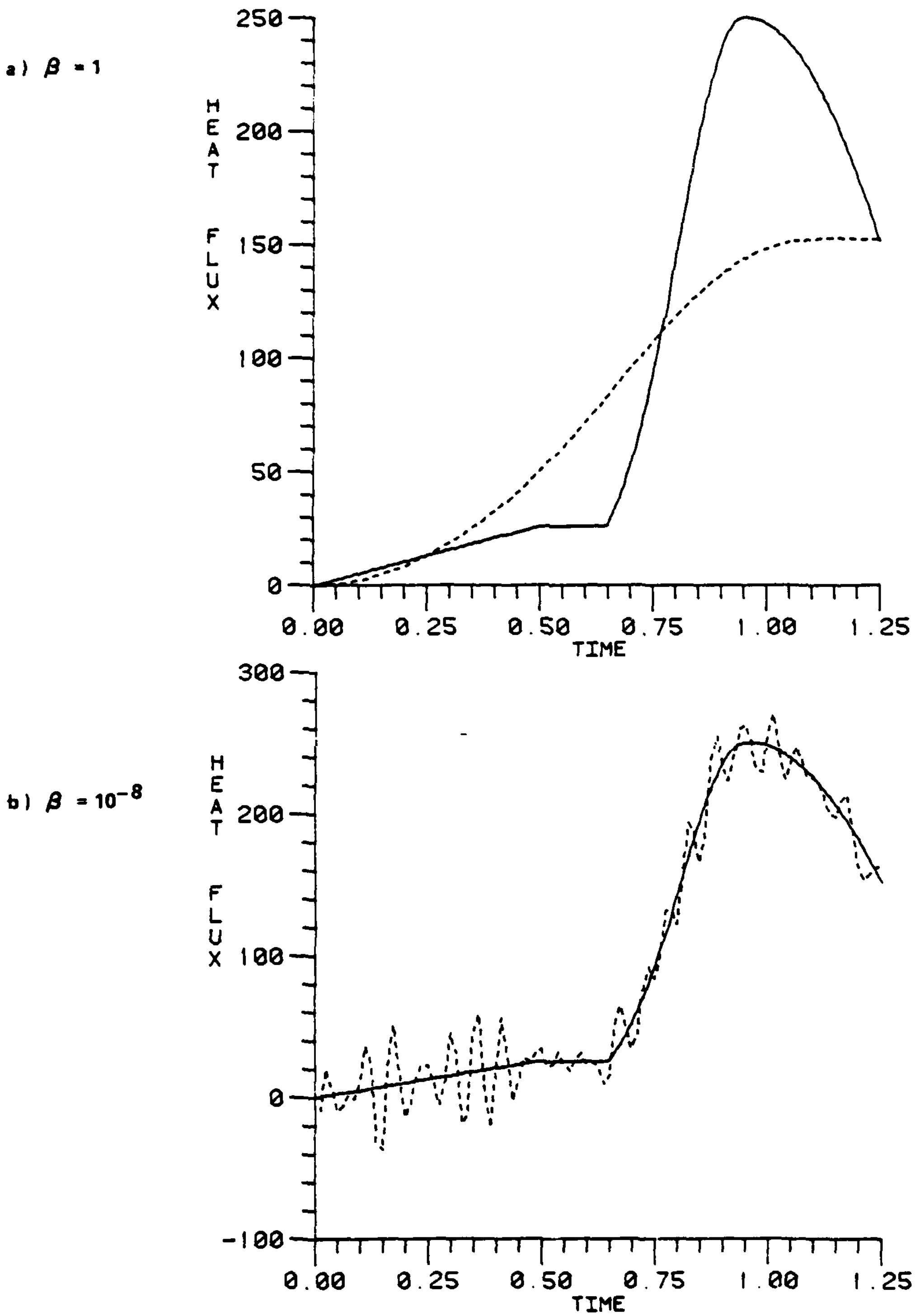

FIGURE 5 APPLICATION OF INAPPROPRIATE β VALUES TO CASE A USING EXPECTED NOISE LEVELS

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APPENDIX A

COMPUTER PROGRAM LISTING

```
      program heatinv(input,output,tape5=input,tape6=output,
     1   tapel1,tapel2)
c
c     this program computes heat fluxes on the front face
c     from temperature data measured at the back face.
c     analysis of several sets of data is possible in
c     each run.
c
c     the data from the back face is read off of tapel1
c     each set of data is assumed to be written unformatted
c     with the number of points first and then the data
c     values.  it is assumed that there are the same number
c     of points in each data set.  otherwise, the matrix
c     must be reformed.
c
c     the program outputs the computed values of qdot (the
c     heat fluxes) are output on tapel2 for post processing.
c     see subroutine output for a description of the write
c     statements
c
c     input of relevant parameters is specified in a namelist.
c
      dimension ip(100)
      common a(100,100),alpha,alam,dx,dt,ulf(100),g(100),b(100)
     1   ,f(101),jj,nt,uinit,aa(3,401),u(401),rhs(401),beta,tmax,fac
c
c   . data stored in blank common:
c
c     a       stores the matrix which approximates the
c             action of the regularized operator which
c             computes backface temperatures from given
c             fluxes on the front face
c     aa      stores the matrix used in solving the heat
c             equation
c     g       stores the vector which specifies the matrix
c     ulf     stores the measured values on the left face
c     f       stores the fluxes that are to be used when
c             solving the heat equation
c     u       stores the temperatures profiles while solving the
c             heat equation
c     rhs     right hand side for solving the heat equation
c     dx      value of delta x used to solve the heat equation
c     dt      value of delta t used to solve the heat equation
c     alam    parameter used for crank nicholson
c     uinit   initial value of the temperature
c     tmax    the length of the run in seconds
c     fac     ratio of u sub x to qdot
```

```
c     nt       number of data points and number of steps taken
c              in solving the heat equation
c     beta     this is the regularization parameter.  beta should
c              be selected so that the estimated 12 norm
c              printed in output correspond to the estimated
c              rms error in the measured data.  when the 12 norm
c              is smaller than the estimate results can become
c              oscillatory. (remedy:  increase beta)  if larger
c              the results can be degraded.  best value of
c              beta is when these two rms values agree.
c              the default value of beta is suitable for the
c              expected noise levels in wind tunnel data reduction
c
      namelist/input/jj,ipr,nt,nthc,beta,uinit,alpha,ak,tmax,delta
c
c     data specified by the namelist
c
c     jj,nt,beta,uinit,alpha,ak,delta are specified above
c
c     alpha        thermal diffusivity in ft**2 per sec
c     ak           thermal conductivity in btu per (ft sec deg f)
c     delta        thickness of the plate in inches
c
      jj=401  $ uinit=0.  $ ipr=1  $ beta=1.e-4
      read(5,input)
      write(6,input)
      mdim=100
      dx=1./float(jj-1)
      dt=1./float(nt)
      alam=alpha*144.*tmax/delta**2*dt/2./dx**2
      fac=12.*ak/delta
      call setmat
      call decomp(nt,mdim,a,ip)
      do 100 i=1,nthc
      read(11)ndp,(ulf(n),n=1,ndp)
      if(ndp.ne.nt) go to 1000
      if(ndp.gt.mdim)go to 1001
      call setrhs
      call solve(nt,mdim,a,b,ip)
      do 300 n=1,nt
      f(n+1)=b(ip(n))
  300 continue
      f(1)=0.
      call output(ipr,i)
  100 continue
      stop
 1000 write(6,1002)i
 1002 format(* number of data points for thermocouple *,i5,
     1 * is inconsistent with the input value*)
      stop
 1001 write(6,1003)mdim,i
 1003 format(* there are more than *,i5,* data points for *
     1  ,*thermocouple *,i5)
```

```
      stop
      end
      subroutine setmat
c
c     this routine computes the matrix corresponding to
c     the normal equations of the regularized approximation
c     to the integral equation
c
      common a(100,100),alpha,alam,dx,dt,ulf(100),g(100),b(100)
     1   ,f(101),jj,nt,uinit,aa(3,401),u(401),rhs(401),beta,tmax,fac
      do 100 n=1,nt
      f(n)=0.
  100 continue
      f(2)=1.  $ f(nt+1)=0.
      call heat(g)
      do 120 id = 1, nt
      n = nt - id + 1
      do 110 jd = 1, id
      m = nt - jd + 1
      a(m,n) = g(jd)*g(id)
      if(jd .gt. 1) a(m,n) = a(m,n) + a(m+1,n+1)
      a(n,m) = a(m,n)
  110 continue
  120 continue
c
c     here we add the stabilization part of the matrix
c
      scr1 = 2.*beta*(dt/3.+1./dt)
      scr2 = beta*(dt/6. - 1./dt)
      ntm1 = nt - 1
      do 200 n = 1, ntm1
      a(n,n) = a(n,n) + scr1
      a(n+1,n) = a(n,n+1) = scr2+a(n,n+1)
  200 continue
      a(nt,nt) = a(nt,nt)+scr1/2.
      return
      end
      subroutine setrhs
c
c     this routine computes the right hand side
c     of the normal equations from the right hand
c     side of the least squares problem
c
      common a(100,100),alpha,alam,dx,dt,ulf(100),g(100),b(100)
     1   ,f(101),jj,nt,uinit,aa(3,401),u(401),rhs(401),beta,tmax,fac
      do 100 i=1,nt
      b(i)=0.
      do 120 j=i,nt
      b(i)=b(i)+ulf(j)*g(j-i+1)
  120 continue
  100 continue
      return
      end
```

```
      subroutine heat(culf)
c
c     this routine computes the solution of the
c     heat equation using crank nicholson
c     the fluxes are specified at x=1 in  the
c     the vector f and the bacface
c     temperatures are returned in the vector culf
c
      dimension culf(1)
      common a(100,100),alpha,alam,dx,dt,ulf(100),g(100),b(100)
     1   ,f(101),jj,nt,uinit,aa(3,401),u(401),rhs(401),beta,tmax,fac
      do 10 j=1,jj
      u(j)=uinit
   10 continue
      call form
      jjml=jj-1
      do 100 n=1,nt
      rhs(1)=(1.-2.*alam)*u(1)+2.*alam*u(2)
      rhs(jj)=(1.-2.*alam)*u(jj)+2.*alam*( u(jjml)+dx*(f(n)+f(n+1)))
      do 110 j=2,jjml
      rhs(j)=(1.-2.*alam)*u(j)+alam*(u(j-1)+u(j+1))
  110 continue
      call trislv(aa,rhs,jj)
      do 120 j=1,jj
      u(j)=rhs(j)
  120 continue
      culf(n)=u(1)
  100 continue
      return
      end
      subroutine form
c
c     this routine forms the matrix used in advancing
c     the heat equation
c
      common a(100,100),alpha,alam,dx,dt,ulf(100),g(100),b(100)
     1   ,f(101),jj,nt,uinit,aa(3,401),u(401),rhs(401),beta,tmax,fac
      do 100 j=1,jj
      aa(2,j)=1.+2.*alam
      aa(1,j)=aa(3,j)=-alam
  100 continue
      aa(1,jj)=aa(3,1)=-2.*alam
      call tridcm(aa,jj)
      return
      end
      subroutine tridcm(a,jj)
c
c     decomposes a tridiagonal matrix stores in band form
c
      dimension a(3,1)
      do 100 j=2,jj
      jml=j-1
      a(1,j)=scr=a(1,j)/a(2,jml)
      a(2,j)=a(2,j)-scr*a(3,jml)
  100 continue
      return
      end
```

```
      subroutine trislv(a,b,jj)
c
c     uses result of tridcm to solve a x = b
c     storing the result in b
c
      dimension a(3,1),b(1)
      do 100 j=2,jj
      b(j)=b(j)-a(1,j)*b(j-1)
  100 continue
      b(jj)=b(jj)/a(2,jj)
      do 200 jd=2,jj
      j=jj+2-jd
      jm1=j-1
      b(jm1)=(b(jm1)-a(3,jm1)*b(j))/a(2,jm1)
  200 continue
      return
      end
      subroutine output(ipr,i)
c
c     computes output for each thermocouple
c
      common a(100,100),alpha,alam,dx,dt,ulf(100),g(100),b(100)
     1   ,f(101),jj,nt,uinit,aa(3,401),u(401),rhs(401),beta,tmax,fac
      write(12)nt,(float(n)*dt*tmax,f(n+1)*fac,n=1,nt)
      if(ipr.eq.0)return
      write(6,1003)i
 1003 format(*1analysis for thermocouple *,i5)
      call heat(b)
      write(6,1000)
 1000 format(*0time*,11x,*backface temperatures(f)*,6x,
     1 ,15x,* heat flux (btu/ft*,2h**,* sec)*/15x,* computed*,7x,
     2   *measured*,7x,*differences*//)
      sum=0.
      do 100 n=1,nt
      t=float(n)*tmax*dt
      sum=sum+(b(n)-ulf(n))**2
      write(6,1001)t,b(n),ulf(n),b(n)-ulf(n),f(n+1)*fac
 1001 format(1x,1p5e15.7)
  100 continue
      sum=sqrt(sum/float(nt))
      write(6,1002)sum
 1002 format(* the 12 norm of differences in the back face*
     1  ,* temperatures is *,1pe15.7)
      return
      end
```

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